V Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme)
(Fresh) (CBCS) (2016 – 17 and Onwards)
MATHEMATICS – VI

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Write Euler's equation when f is independent of y.

b) Show that the functional $I = \int_{x_1}^{x_2} (y^2 + x^2 y^1)\,dx$ assumes extreme values on the straight line y = x.

c) Define geodesic on a surface.

d) Evaluate $\int_C (5x\,dx + y\,dy)$ where C is the curve, $y = 2x^2$ from (0, 0) to (1, 2).

e) Evaluate $\int_0^{\pi} \int_0^{\sin y} y\,dx\,dy$.

f) Evaluate $\int_0^1 \int_0^x \int_0^z dy\,dz\,dx$.

g) Show that the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab using Green's theorem.

h) Evaluate using Stoke's theorem $\oint_C (yz\,dx + zx\,dy + xy\,dz)$ where C is the curve $x^2 + y^2 = 1,\ z = y^2$.

PART – B

Answer **two full** questions : (2×10=20)

2. a) Prove that the necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')\,dx$ with $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

b) Find the geodesic on a plane.

OR

3. a) Show that the extremal of $I = \int_{x_1}^{x_2} \sqrt{y(1+(y')^2}\,dx$ is a parabola.

b) Find the extremal of the functional $I = \int_0^1 \sqrt{1+(y')^2}\,dx$ with $y(0) = 1$ and $y(1) = 2$.

4. a) Find the shape of a chain which hangs under gravity between two fixed points.

b) Find the extremal of the functional $\int_0^1 \left[(y')^2 + x^2\right]dx$ subject to constraint $\int_0^1 y\,dx = 2$ and having end conditions $y(0) = 0$, $y(1) = 1$.

OR

5. a) Find the function y which makes the integral $I = \int_{x_1}^{x_2} [y^2 + 4(y')^2]\,dx$ an extremum.

b) Find the extremal of the functional $I = \int_0^{\pi} \left[(y')^2 - y^2\right]dx$ with $y(0) = 0$ and $y(\pi) = 1$ and subject to the constraint $\int_0^{\pi} y\,dx = 1$.

PART – C

Answer **two full** questions :

(2×10=20)

6. a) Evaluate $\int_C (x+y+z)\, ds$ where C is the line joining the points (0, 1, 0) and (1, 2, 3).

b) Evaluate $\iint_A (4x^2 - y^2)\, dxdy$, where A is the area bounded by the lines y = 0, y = x and x = 1.

OR

7. a) Evaluate $\int_0^{\infty}\int_0^{\infty} x\, e^{x^2/y}\, dx\, dy$, by changing the order of integration.

b) Find the area bounded by the arc of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in first quadrant.

8. a) Evaluate $\int_0^1 \int_0^{x^2} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{1-x^2-y^2-z^2}}$.

b) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\, dxdy$ using polar co-ordinates, where R is the annular region between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 1$.

OR

9. a) Find the volume bounded by the surface $z = a^2 - x^2$ and the planes x = 0, y = 0, z = 0 and y = b.

b) If R is the region bounded by the planes x = 0, y = 0, z = 0 and x + y + z = 1, show that $\iiint_R z\, dxdydz = \frac{1}{24}$.

PART – D

(2×10=2[illegible]

Answer **two full** questions :

10. a) State and prove Gauss' Divergence Theorem.

b) Evaluate using Green's theorem for $\oint_C [xy\,dx + yx^2 dy]$, where C is the curve enclosing the region bounded by the curve $y = x^2$ and the line $y = x$.

OR

11. a) Verify Green's theorem in the plane for $\oint_C \left[(x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy\right]$, where C is the square with vertices (0, 0), (2, 0), (2, 2) and (0, 2).

b) Evaluate $\iint_S \vec{F}\cdot\hat{n}ds$ using divergence theorem where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the closed surface bounded by planes z = 0, z = 1 and the cylinder $x^2 + y^2 = 4$.

12. a) Verify Stokes theorem for $\vec{F} = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$; C is the boundary of the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 9$.

b) Evaluate using Gauss' divergence theorem $\iint_S \vec{F}\cdot\hat{n}ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes x = 0, y = 0, z = 0, x = 1, y = 2, z = 3.

OR

13. a) Evaluate $\oint_C \vec{F}\cdot d\vec{r}$, using Stoke's theorem where $\vec{F} = (y - z + 2)\hat{i} + (yz)\hat{j} - (xz)\hat{k}$ taken over the surface S of the cube $0 \le x \le 2,\ 0 \le y \le 2,\ 0 \le z \le 2$.

b) By using Green's theorem evaluate $\oint_C [(3x - y)\,dx + (2x + y)\,dy]$ where C is the circle $x^2 + y^2 = a^2$.
